Activity 1 proof

1. Prove that any square number when divided by 3 leaves a remainder of 0 or of 1

Hint: Any number can be written in the form $3n,\ \ 3n \pm 1 \ \ n \in \mathbb{N}$

2. Prove that any square number when divided by 5 leaves a remainder of 0 or of 1 or of 4

This could also be done by exhaustion.

3. Factorise $x^2 - y^2$

Show that any odd number can be written the difference of the squares of consecutive numbers.

e.g. $7 = 4^2 - 3^2$, $55 = 28^2 - 27^2$

4. Prove by exhaustion that the equation $x^2 + y^2 = 24$ has no solutions for $x, y \in \mathbb{N}$

Give an estimate for the maximum number of cases that you would have to consider to show the equation $x^2 + y^2 = k$ where $k \in \mathbb{N}$ has no solutions for $x, y \in \mathbb{N}$

5. Prove, by exhaustion, that the number of factors of 512 is 10

How can the work required be reduced? (A key point in computer assisted proofs)

6. Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x}$ prove, by exhaustion that y cannot be of the form x^n

7. Prove, by exhaustion, that the equation $2x^2 + 5y^2 = 8004$ has no solutions for $x, y \in \mathbb{N}$

8. Prove that $p(n) = n^2 + n + 17$ is **not** always prime for all values of $n \in \mathbb{N}$

9. Let $F_n = 2^{2^n} + 1$ Show that F_n is prime for n = 0, 1, 2, 3, 4.

Use a factoriser to show that F_5 is NOT prime.

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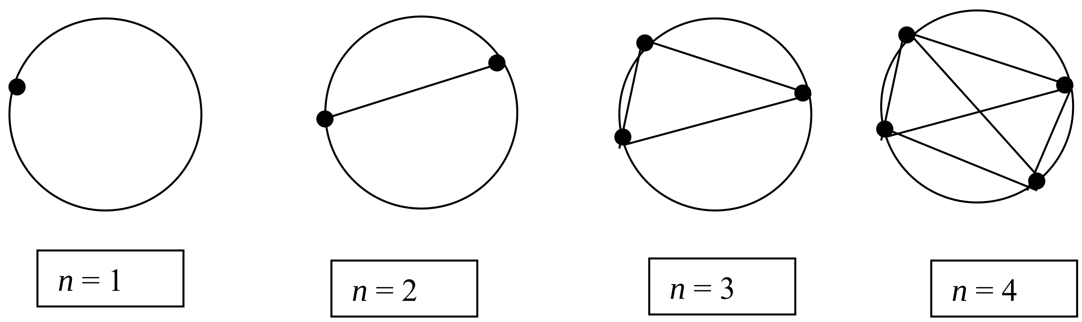

Based on the diagrams make an obvious conjecture about how the number of regions in the disc depends on the number of points marked on the circle.

Show, using a counter example, that your conjecture is wrong.

11 I claim that any natural number greater than 1 can be written as the sum of consecutive natural numbers.
For example $3 = 1 + 2$ $12 = 3 + 4 + 5$

Give a counterexample to show my claim is wrong.

Suggest a different claim based on (at least 2!) counterexamples.

12 (a) Give a counterexample to show that $\sin(2x) \neq 2(\sin x)$

(b) Give a counterexample to show that $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$

***13** Prove that the equation $2x^2 + kx + k^2 = 0$, $k \neq 0$ has no real roots in x

***14** For each statement below you must state if it is always true, sometimes true or never true, giving a reason in each case.

(a) The x - axis is a tangent to the curve $y = (x-a)^2$ for all values of a

(b) $\frac{a}{x} > b \Rightarrow bx < a$

(c) The sum to infinity of the series $1 + r^2 + r^4 + r^6 + ...$ is $\frac{1}{1-r^2}$

* based on the home GCE specimen papers